Premia v3 Specification

Litepaper

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June 29, 2023

Abstract

Premia v3 is a non-custodial options settlement engine and automated market maker (AMM) implemented for the Ethereum Virtual Machine. This version of the protocol implements a base layer exchange that enables the permission-less creation of option pools. Each pool utilises concentrated liquidity, partial collateralisation, pro-rata fee growth for liquidity providers, and integrations for strategic vaults and quote systems. These innovations improve the capital efficiency, composability, and sustainability of the protocol. This lite version of the Premia v3 whitepaper [2] outlines it's major components and features.

1. The exchange

The exchange supports buying and selling of call and put options. A call (put) option is a financial asset that gives the option holder the right, but not the obligation, to buy (sell) an underlying asset at a pre- determined date T, the maturity date, and a pre-determined price K, known as the option's strike.

1.1. Full collateralisation and quoted prices

To minimize counterparty risk on the exchange, options are fully collateralised. Call options are secured by the underlying and put options are backed by the strike amount in quote tokens (eg. USDC). Due to the introduced capital inefficiencies, lending pools are introduced in section 2, which can enable partially collateralize positions. Prices of call options are quoted in the underlying and prices of puts are quoted in the quote token (most often US-dollar stablecoins, e.g. USDC). Therefore call prices are range-bound to the price range $0 \leftrightarrow 1$ (quoted in the underlying) and put prices are bound to $\$0 \leftrightarrow \$K$.

1.2. Concentrated liquidity

Liquidity providers (LPs) are able to create positions in specific option pools as concentrated range orders with defined lower and upper price bounds. Each range order belongs to a single pool for a specific option (combination of strike price, maturity date, and option type). This enables active traders with high conviction to maximise fee collection from highly capital efficient, concentrated orders, while passive LPs with less conviction can still earn trading fees in a wider, less capitally efficient range. Option positions are represented as fungible transferable ERC-1155 tokens.

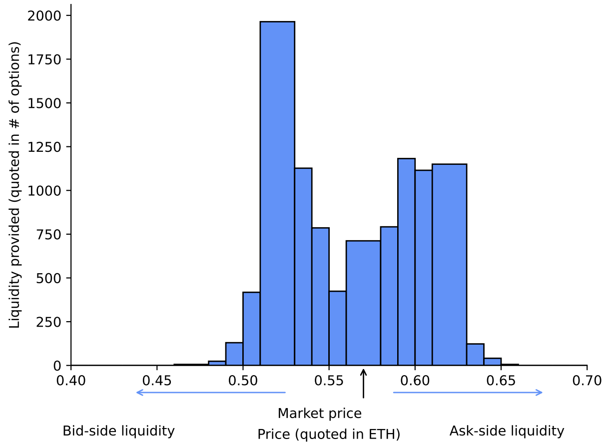

Figure 1: Exemplary liquidity distribution of a Premia v3 call option pool featuring concentrated liquidity.

1.3. Range orders

Contrary to Uniswap v3 [1] where a single range order is sufficient to support a swap market, in an options market, two distinct range orders are necessary to enable LPs to

Figure 2: Liquidity profile of a CS range order deposited above the market price. As liquidity takers come to the exchange and buy long options the price traverses through the range order such that the LP gains short exposure. Once the market price is at the upper tick price (0.22 ETH) the range order is filled. A full withdrawal would result in the LP obtaining 0.63 ETH and 3 short positions.

gain or reduce long or short exposure:

Collateral-Short (CS) range orders Collateral and / or short contracts can be deposited within the range order to increase or reduce short exposure. The mix of collateral and short contracts initially deposited is dependent upon the market price. When liquidity is deposited above the market price, the deposit is entirely composed of collateral which is used to underwrite options, i.e. as liquidity takers (LTs) buy longs the market price increases and the order is traversed (see Figure 2). Contrary, liquidity deposited below the market price is comprised of short contracts, which will be bought-to-close and unlock the remaining stored collateral. A range order that straddles the market price comprises a mix of collateral and shorts.

Long-Collateral (LC) range orders Collateral and / or long contracts can be deposited within the range order to increase or reduce long exposure. Liquidity deposited above the market price is entirely in long contracts which are sold to reduce long exposure (see Figure 3). Contrary, liquidity deposited below the market price is entirely composed of collateral to buy-to-open long contracts (see Figure 4). A market order straddling the market price consists of a mix of both assets.

We explain to the technical reader the corresponding equations that define the liquidity composition of a CS and LC order given any market price p in Appendix A.

1.4. Linear relationship of price and trade size

The majority of the AMMs deployed in decentralised finance (DeFi) are so-called constant function market markers (CFMMs). In contrast, the exchange presented here does not employ a trading function that governs the exchange of assets. Instead a linear relationship of price and liquidity is specified which is feasible due to the range-bounded nature of option prices. More precisely, if we assume that the price range $a \leftrightarrow b$ is an active tick range with liquidity L and let the current price be $p \leftarrow a$, then for any positive trade size within the active tick range $x \in [0, L]$ the price is defined as

$$p(x) = b + (b - a)\frac{x}{L} . \tag{1}$$

Example 1 (Buying options)**.** Assume we are in the situation of Figure 2: 3 units of liquidity are deposited within the range $0.2 \leftrightarrow 0.22$ ETH and assume the market price is at 0.2 ETH. If an LT buys 1.5 long contracts the market price will move to 0.21 since half of the order is filled and the exchange is governed by (1). The quoted price per option is $\frac{0.2+0.21}{2} = 0.205$ ETH. In summary, the LT will receive 1.5 long contracts in return for $1.5 \cdot 0.205 = 0.3075$ ETH.

1.5. Accounting: pro-rata distribution of fees

Similar to Uniswap v3, the exchange utilise a tick-based system that allows distributing the fee income on a pro-rata basis. For this purpose, the *liquidity provided per tick* is tracked as a global state variable. We refer to Section 5 of the whitepaper [2] for a full presentation of the state variables that are tracked.

2. Margin: obtaining leverage

On the exchange level, all options that are underwritten are fully collateralised, thus, minimising counterparty and insolvency risk. However, full collateralisation creates capital inefficiencies and limits the space of trading strategies that LPs and LTs can deploy. To support market participants with a rich space of trading strategies lending pools are introduced that allow LPs and LTs to borrow and partially collateralise their short option contracts.

2.1. Lender capital

Lenders can deposit their capital into a lending pool and earn an interest depending upon the utilisation. Each lender must select a deadline on which their capital is returned. Any lender capital utilised in an option position is locked for up to the expiration of the position.

Figure 3: Liquidity profile of an LC order deposited above the market price in a put option pool. As the market price traverses the range order longs are sold to the LT and in return premiums are transfered. After full traversal the position's liquidity is composed solely of the quote asset.

Lenders can withdraw their locked (utilised) capital at any time, provided there is sufficient available capital in the pool. In this case, the withdrawing lender must pay a commitment fee to borrow the capital from the other lenders in the pool, in order to unlock their locked capital.

2.2. Borrowers

Upon borrowing, a borrower pays an upfront commitment fee based on the utilisation of the total available capital in the pool. Whenever an LP / LT borrows they are in a first-loss position for the exposure taken on. This means that the borrower's collateral is first used to cover any loss if a position becomes unprofitable. All profit is retained by the user borrowing capital; less capital usage fees.

Example 2. Assume a user borrows 0.8 ETH, uses 0.2 ETH from his own funds to achieve full collateralsation and underwrites an option for 0.3 ETH. If the exercise value of the option is 0.15 ETH the borrower will receive during settlement 0.3 + (0.2 - 0.15) = 0.35 ETH.

In case a short contract is closed prior to expiration, borrowers can close borrowed positions early and receive a rebate on commitment fees paid. Furthermore, borrowers can refinance their position if the minimum margin requirement (discussed in the subsequent section) increases or decreases.

2.3. Margin requirements

A borrower has to fulfil margin requirements in order to be able to borrow and not get liquiditated over the lifetime of an option. The *minimum margin* amount is a threshold value, dynamic over the lifetime of an option, which determines a position's point of liquidation. It is based on the 5% value-at-risk (VaR) for a single-tail that admits the option market's current implied volatility as an input value, which is queried from a volatility oracle that is based on an eSSVI model. The minimum margin requirement to avoid a liquidation during the lifetime of an option is defined as the 95% VaR bounded below by 3% of the notional value of the position. To open a margin position the borrower has to meet the *initial margin requirement* and deposit 1.5 times the minimum margin requirement (see Figure 5). The exact formula to compute the minimum and initial margin requirement can be found in the Appendix B.

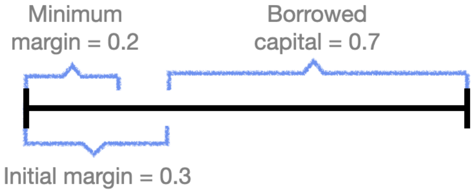

Figure 5: Example scenario of minimum and initial margin requirements upon borrowing.

Figure 4: Liquidity profile of an LC range order deposited below the market price. Given the pool's market price crossed the upper tick price (0.22 ETH) a further decrease in the market price increases the long exposure of the LP as the LP buys with collateral long contracts.

Figure 6 exemplifies the development of the minimum margin requirement for a 40-day ATM call option under the assumption of a Black-Scholes model with 100% volatility. Minimum margin increases / decreases with spot and overall decreases as a function of time since minimum margin was defined as the VaR of the option offset by the fair-value at time of underwriting the option.

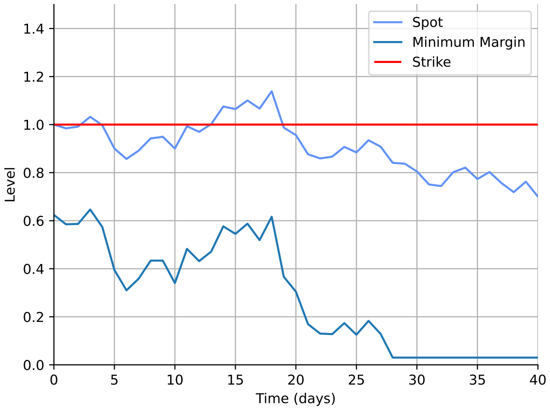

Figure 6: Exemplary development of the spot price and the minimum margin requirements.

3. OTC Liquidity

Market-makers on-chain have to optimise between active and passive liquidity management, factoring EVM transaction fees into their calculations. The over-the-counter (OTC) quote system enables vaults and market-makers on- or off-chain to provide quotes to users or aggregators, which can then be fulfilled through the exchange alongside range orders. This increases liquidity across strikes and maturities and enables professional market makers to optimise for both execution price and transaction fees. However, these market makers will need to source their own trading volume as off-chain quotes will not yet be displayed to users directly interacting with a pool's smart contract. In addition, OTC orders do not receive maker rebates, rather, the full taker fee is retained by the protocol.

References

[1] H. Adams, N. Zinsmeister, M. Salem, R. Keefer, and D. Robinson. Uniswap v3 core. *Uniswap, Tech. Rep.*, 2021. URL `https://uniswap.org/whitepaper-v3.pdf`.

[2] Froggie, Enso, Wolfy, T. Guceri, and M. Wiese. Premia v3 Specification. 2023. URL `https://premia.finance/v3.pdf`.

A. Range order math

By convention, the liquidity deposited into a range order will be denoted by L and quoted in terms of the option contracts. This number will also represent the amount of fungible LP tokens that will be minted whenever a CS / LC order is created. For clarity, we first only consider call option pools.

Call option pools

COLLATERAL-SHORT LP POSITIONS

In case of a CS order, the amount of collateral deposited translates directly into the liquidity deposited, since one short option contract has to be secured by one unit of the underlying (full collateralisation). Assuming the LP holds L CS order tokens for the range $a \leftrightarrow b$, the assets he owns at any price $p \in [0, 1]$ is governed by the equations:

$$\text{collateral}^{\text{CS}}_{\text{call}}(p) = \begin{cases} L & p < a \\ L\,(\frac{b-p}{b-a} + \frac{p^2-a^2}{2(b-a)}) & a \leq p \leq b \\ L\,\frac{a+b}{2} & p\ > b \end{cases}$$

$$\text{short}(p) = \begin{cases} 0 & p < a \\ L\,\frac{b-p}{b-a} & a \leq p \leq b \\ L & p\ > b \end{cases}$$

Thus, if an LP deposits above the market price, the position initially is solely composed of L units of collateral. During traversal, the collateral is used to underwrite options and simultaneously premiums are collected from selling the options. After traversal, the position consists of premiums and L short contracts. This is exemplified in Figure 2 where $L = 3$ units of collateral were deposited within the range $0.2 \leftrightarrow 0.22$ ETH.

If an LP deposits below the market price, the LP deposits L short positions and additionally the collateral to cover the premiums for buying-to-close the short contract. During traversal, the amount of shorts decays linearly and the collateral is used to pay the premiums. After traversal, the order solely consists of the unlocked from covering the shorts collateral.

LONG-COLLATERAL LP POSITIONS

LC orders have similar dynamics to CS orders. However, here collateral is deposited below and long options are deposited above the market price to increase / reduce long exposure. Furthermore, as opposed to CS orders, one unit of collateral translates to $\frac{2}{a+b}$ liquidity tokens, since one unit of collateral can buy this amount of long contracts.[1] Given a LC range order with range $a \leftrightarrow b$, liquidity L the asset composition for any price $p \in [0, 1]$ is defined as:

$$\text{collateral}^{\text{LC}}_{\text{call}}(p) = \begin{cases} 0 & p < a \\ L\,\frac{p^2-a^2}{2(b-a)} & a \leq p \leq b \\ L\,\frac{a+b}{2} & p\ > b \end{cases}$$

$$\text{long}(p) = \begin{cases} 0 & p < a \\ L\,\frac{b-p}{b-a} & a \leq p \leq b \\ L & p\ > b \end{cases}$$

If an LP deposits liquidity into a LC range order above the market price, the liquidity will encompass solely long options, which will be sold when the market price traverses through the range order. Once the market price has fully traversed the range order, the range order only consists of collateral, which equals the amount of options deposited times the average price (the fill price) of the range orders range.

Contrary, if an LP deposits liquidity into an LC order below the market price, the order consists solely of collateral which is used to buy long options from LTs when they want to sell their long contracts. When the market price traverses the range order from right to left collateral is used to buy long options contracts or buy-to-close short contracts of traders. Once it is fully traversed the liquidity composition solely consists of long contracts.

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[1] Assume the average price of a range order is 0.2, then one unit of collateral can buy 5 long contracts.

Put option pools

As noted in subsection 1.1 prices for put options are quoted in dollar amounts. The price range for a put option therefore is $[0, K]$; the dollar strike being the maximum value a put option can have. A short option contract is secured by K dollars. Therefore, when L short option contracts are closed, KL dollars are released. Given any put price $p \in [0, K]$ the collateral a CS / LC LP position holds is defined as:

$$\text{collateral}^{\text{CS}}_{\text{put}}(p) = K\,\text{collateral}^{\text{CS}}_{\text{call}}(pK^{-1})$$
$$\text{collateral}^{\text{LC}}_{\text{put}}(p) = K\,\text{collateral}^{\text{LC}}_{\text{call}}(pK^{-1})$$

The equations for short and long both hold also for put option pools.

B. Margin

The Value-at-Risk of an option at time t sold for fair value at time t^* is defined at the α confidence level as

$$\text{VaR}^{\alpha}_{t}(S_t) = -FV_{t^*} + \begin{cases} \left(S_t\,\exp(z_{1-\alpha}\sigma_t\sqrt{\tau} - \frac{1}{2}\sigma_t^2\tau) - K\right)^+ & \text{if call} \\ \left(K - S_t\,\exp(-z_{1-\alpha}\sigma_t\sqrt{\tau} - \frac{1}{2}\sigma_t^2\tau)\right)^+ & \text{if put} \end{cases}$$

where FV_{t^*} denotes the option's fair value at the time of borrowing the capital (t^*), σ_t denotes the option's implied volatility at time $t \in [t^*, T]$, which is queried from the volatility oracle, and $z_{1-\alpha}$ is the $1 - \alpha$ quantile of the standard normal distribution. From the VaR we can define the minimum margin threshold, which is defined as the VaR normalized by the spot price for call options and floored at $c = 3\%$ of the full collateralisation level

$$M_t = \begin{cases} \max\left(c, \min\left(\frac{\text{VaR}_{\alpha}(S_t)}{S_t}, 1\right)\right) & \text{if call} \\ \max\left(cK, \min\left(\text{VaR}_{\alpha}(S_t), K\right)\right) & \text{if put} \end{cases}$$

Using the minimum margin the initial margin is defined as the minimum margin capped at 1 for call and K for put options

$$M^*_{t^*} = \begin{cases} \min\left(rM_t, 1\right) & \text{if call} \\ \min\left(rM_t, K\right) & \text{if put} \end{cases}$$